# André Lévêque's Contribution
# A Review of His Linear Velocity Profile Approximation

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## 1 Introduction

In 1928, André Lévêque defended his PhD thesis in Paris. Its title was *Les Lois de la Transmission de Chaleur par Convection* - *The Laws of Convective Heat Transfer* - and it was published that same year in *Annales des Mines*, the famous French mining engineering journal [3]. Lévêque's contribution to engineering was to show people how to think in a new way about how heat moves across a thin layer of fluid close to a wall.

## 2 The Poiseuille Problem

Most researchers who know of André Lévêque do so because of an idea he presented on p285 of his thesis, about heat transfer close to walls in certain *Poiseuille* flows [3]. A Poiseuille flow is characterised by laminar flow through a pipe or a channel. According to Schlichting [6], Lévêque,

> introduced the very reasonable assumption that the whole of the temperature field is confined inside that zone of the velocity field where the longitudinal velocity component $u$ is still proportional to the transverse distance $y$.

That is, the velocity profile is approximated as being linear very close to the surface. This was true only for Poiseuille flows of large Prandtl number,

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where the fluid temperature changes faster with distance $y$ from a hot wall than the fluid velocity does. Lévêque demonstrated this as follows. In a Poiseuille flow, the actual velocity profile is parabolic and takes the form [3, 4, 1]:

$$u(y) = u_0\left(1 - \frac{(h-y)^2}{h^2}\right) \tag{1}$$

where $u_0$ is the maximum velocity, at the centre of the channel, $y$ is the distance normal to the wall, and $h$ is the half-height of the channel or the radius of the pipe, where the velocity is $u_0$. By rewriting this last equation in terms of $y/h$,

$$u(y) = u_0\frac{y}{h}\left(2 - \frac{y}{h}\right) \tag{2}$$

Lévêque [3, pp 284-287] observed that for flows of large Prandtl number, convective heat transfer is affected only by the velocity values very close to the surface of the pipe. In this region, $y/h$ is small and,

$$u(y) = 2u_0\frac{y}{h} \approx \beta y \tag{3}$$

Where $\beta y$ is the wall tangent of the fully developed parabolic velocity profile. With this simplification for $u(y)$, which we can name for him, Lévêque arrived at an asymptotic solution for high Prandtl number heat transfer into a fully developed Poiseuille flow [4]. It is the simplification of the velocity profile that Lévêque is remembered for.

## 3 The Boundary-Layer Problem

### 3.1 Lévêque

Schlichting's reference to André Lévêque [6, Chap. 12, p291] needs qualification. Lévêque did not solve a thermal boundary-layer problem; his solution was specific to heat transfer into a Poiseuille flow [5]. In this type of flow, $u$ is a function of $y$ only, it does not change with streamwise location $x$.

### 3.2 Schuh

Schuh [8, 7] observed that in a boundary-layer, $u$ is again a linear function of $y$, but that in this case, the wall tangent is a function of $x$, the distance along the wall. He expressed this with a modified version of Lévêque's profile, $u = \beta(x)y$, and used this linear velocity profile to tackle the problem of heat-transfer across laminar boundary-layers. Schuh does not reference Lévêque in his paper but he had the same insight as Lévêque did in 1928, explaining:

> On inspection of some solutions ... the thickness of the thermal boundary layer is found to be become small compared to the velocity boundary layer if either $n$ or $Pr$ (Prandtl number) becomes large. Then it is sufficient to replace the velocity profile by its tangent at the wall, since, for calculating the temperature field, only that part of the velocity profile is of influence that lies within the thermal boundary layer.

Lévêque and Schuh's simplifications work for large Prandtl number or in any situation when the momentum boundary-layer thickness is greater than the thermal boundary-layer thickness. This also works well for low Prandtl numbers, though not when the Prandtl number is very much less than one [4].

### 3.3 Kestin-Persen

Kestin and Persen's 1962 paper was wider in scope, describing large Prandtl number solutions for many different wall temperature distributions [2]. Schuh's work considered a single distribution. Without referencing Schuh or Lévêque, Kestin and Persen state (p357):

> The second simplification consists in the fact that the variation of $u$ with $y$ is linear.

and, like Schuh, they write this in the form $u = \beta(x)y$. For the problem of a flat plate with a temperature jump at $x = x_0$, they propose a substitution that reduces the parabolic thermal boundary-layer equation to an ordinary differential equation. The solution to this equation, the temperature at any point in the fluid, can be expressed as an incomplete gamma function.

### 3.4 Schlichting

After Kestin and Persen, and crediting Lévêque and Schuh, Schlichting proposes an equivalent substitution that reduces the thermal boundary-layer equation to an ordinary differential equation whose solution is the same incomplete gamma function [6, Chap. 12, Thermal boundary layers in laminar flow, p291, Eq. 12.60].

## 4 Conclusion

### 4.1 Lévêque's Technical Contribution

André Lévêque seems to have been the first to observe that when the transition from surface to freestream temperature takes place across a very thin region close to the surface, the most important fluid velocities, those inside this very thin region, change linearly with normal distance from the surface (i.e. $u = \beta y$, where is the wall tangent). Schuh also saw this and, in 1953, showed how to apply this idea to boundary-layers, with the modification that the wall tangent is a function of $x$, $u = \beta(x)y$. Kestin and Persen come to this idea independently, outlining with clarity the solution that Schlichting describes in Boundary-Layer Theory. This solution, of the thermal boundary-layer equation for flows of large $Pr$, appears to be Kestin and Persen's, not Lévêque's.

### 4.2 A Brief Sketch of Lévêque's Life

André Marcel Lévêque in 1896 in Beauvais to Henri and Blanche Lévêque. He fought throughout the First World War, becoming an officer and earning a *Croix de Guerre*. After the war he studied at the École Polytechnique and at the École des Mines. He married Clotilde Foret in 1925 in Béthune. He presented his doctoral thesis in 1928. He had two children; one, a daughter, died aged three, in the same year as his thesis defence. His son, Jean, was born in 1929. André Lévêque died tragically from tuberculosis in 1930, aged 33. He is buried in Beauvais[1].

## References


[1] J. A. Cooper and R. G. Compton, *Channel electrodes - a review*, Electroanalysis, 10 (1998), pp. 141 – 155.

[2] J. Kestin and L. Persen, *The transfer of heat across a turbulent boundary layer at very high prandtl numbers*, Int. J. Heat Mass Transfer, 5 (1962), pp. 355–371.

[3] A. Lévêque, *Les lois de la transmission de chaleur par convection*, Annales des Mines ou Recueil de Mémoires sur l'Exploitation des Mines et sur les Sciences et les Arts qui s'y Rattachent, Mémoires, Tome XIII (13) (1928), pp. 201 – 239.

[4] H. Martin, *The generalized lévêque equation and its practical use for the prediction of heat and mass transfer rates from pressure drop*, Chemical Engineering Science, 57 (16) (2002), pp. 3217–3223.

[5] ——, *Personal communication*, 2005.

[6] H. Schlichting, *Boundary-Layer Theory*, McGraw-Hill, New York, 7th ed., 1979.


[1]This biographical information was compiled by Holger Martin with the assistance of Jean Lévêque and the local authorities of Beauvais and Béthune. The original biography, with more detail, is freely available at my own website and we published the same material to Wikipedia.

[7] H. Schuh, *On asymptotic solutions for the heat transfer at varying wall temperatures in a laminar boundary layer with Hartree's velocity profiles*, Jour. Aero. Sci., 20(2) (1953), pp. 146–147.

[8] ——, *A new method for calculating laminar heat transfer on cylinders of arbitrary cross-section and on bodies of revolution at constant and variable wall temperature*, TN 33, KTH Aero, 1954.